Should gold and silver be regarded as two separate asset classes? New insight using threshold cointegration and volatility spillover methodologies

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Abstract

The paper makes an attempt to examine whether Gold and Silver be considered as single or two separate asset classes while considering asset allocation and portfolio diversification. To this end the paper first analysed the risk-return profile of the two assets and later developed a co-integration and volatility spillover models to understand the characteristics and dynamic movement of the two precious metals. For empirical analysis, daily closing spot prices were collected for ten year period; April 1, 2012-March 31, 2022 for the two precious metals from the MCX Exchange of India. The results of the study failed to detect any long run cointegration using two different tests viz. the Threshold and Johansen cointegration techniques, the former also included structural break in time series. Two cointegration tests were necessitated after the BDS tests showed non linearity in the movement of both the variables under study. Short run causality for which VAR Granger test was employed was however noticed under the study with Gold causing movement in Silver but not vice versa. The volatility spillover was also seen moving from gold to silver from the results. The data description revealed vast differences in risk and

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return profile of the two assets. The study also tested for cointegration model pre-requisites like Serial Correlation, Stability and Heteroscedasticity and found these to be satisfactory. The study therefore concludes that in the short run Gold appears to be influencing the movement of Silver, however in the long run no relation between the two variables was visible from study results. Hence Gold and Silver may be considered as separate assets by the investors and portfolio managersas a part of asset allocation strategy if the viewpoint of the manager is long run.

Keywords: Threshold Co-integration Model, Volatility Spillover, Structural Break, Non-Linearity, VAR Causality.

1. Introduction

Precious metals which primarily include Gold and Silver have been traditionally viewed as instruments of store of monetary value. This is mainly because they possess unique twin characteristics which makes them score over other metals; first is malleability which makes these metals flexible enough to be transformed to any shape and size and second being the non corrosiveness property which makes them strong and non-reactive when in contact with other substances. Out of the two, gold has been first choice amongst monetary authorities for decades and the biggest example has been the gold system and gold exchange system of paymentswhich were globally adopted under the Bretton wood system. On the other hand, Silver was included by Germany in its monetary system during the 19th century (Schweikert, 2018).

Apart from being used as a monetary asset, gold has also served the investing class by acting as a ‘safe haven’ by maintaining its value during times when other prominent assets like equities and bonds have witnessed a correction or fall in their values, largely seen during the crisis periods. Thus, when most assets including stocks, bonds and real estate move in tandem towards a downward journey during uncertain times, gold maintains its value (Chua, 1990). This ‘yellow metal’ has also been a popular inflation hedge instrument amongst the investors. Silver, on the other hand, has some significant commercial applicationsdue to its special reflective characteristics. The 'white metal' has therefore found usage inindustries likephotography, optics, batteries, switches and coating material.

Perhaps, the single largest industry to consumeboth gold and silver is the jewelry and ornamental industry. According to an estimate, out of the total world demand for gold, approximately 50 % comes from this industry while for silver; this demand is 30 % of global demand (www.mcxindia.com). The market where such a demand exists is primarily located in Asian Region. In some Asian economies, it is customary to wear some ornament or jewellery and out of the two, the first choice of people is gold followed by silver, silver has traditionally been viewed as poor man's gold.

On the other hand, it is quite surprizing that although being an instrument of monetary value for centuries, empirical studies on gold and silver especially the ones which consider this asset from investment perspective is fairly of recent origin.One of the probable reasons why precious metals had managed to escape the empirical analysts radar is that these metals are considered both as a commodity and also as a financial asset. Many researchers even today believe that the pricing of gold is not still fully understood. Now, whereas asset like equities are typically priced on the basis of earnings which get converted to dividends, bonds are priced on the basis of macro movements, economic conditions and risks of issuer, the precious metals on the other hand being both a commodity for consumption and also as a financial asset; follow no consistent pricing model. If we compare gold and silvertoanother commodity, say crude which is globally the most tradeable commodity, then there appears to be a striking difference in price determinants of the two; whereas crude is strongly influenced by annual production , the same is not true for the gold and silver whose stock position many times have far exceeded their production. The main determinant of gold according to most economists is its demand, which again is not from industry but from ornaments and jewellery thereby making it a proxy for store of value. Hence this makes gold a simply a cash store asset. On the other hand, for silver, prices are somewhat determined by industrial demand and therefore a proxy for store of value appears to be only partially true.

The empirical research on gold and silver has gathered some momentum during the last two decades and broadly speaking we can classify this research into five categories or types; studies under the first type explore the ability of precious metals to play the role of a risk diversifier i.e. their capability to stand crisis without eroding its value. The researchers have employed different tools and techniques to test the ability of these precious metals as an instrument of hedge or safe haven under different market conditions and results have been positive and encouraging (Shahani and Bansal, 2021; Dee and Zheng, 2013; Baur and Lucey, 2010). Thus, considering the current scenario where there are not enough asset classes available for risk diversification during crisis periods and with crisis in markets becoming a feature of quick succession, a lot of market participants, tend to include gold and silver in addition to other assets in their portfolios. Jaffe (1989) who was the one of the early researchers in this field showed that the investors can substantially reduce their risk by including just 5 % gold in their portfolio.

The second category of empirical research on gold and silver pertain to the angle of price discovery i.e. whether or not futures tend to predict the spot prices in case of precious metals and some of the studies which have focused on this dimension are Jin et al., (2018); Kumar and Arora, (2011) amongst others. Studies broadly have concluded that like other markets in case of precious metals too, it is the futures market which tends to discover the price. The third dimension pertaining to gold and silver explores the market efficiency of these precious metals and most research papers under this category have focused on the seasonality in the movement of their prices (day, week or month of the year effect). The seasonality which has been detected in a lot of research studies with respect to 'yellow metal' pertain to either 'Monday Effect' or the 'autumn effect'. An interesting observation regarding these two effects was that these effects tend to vanish during the crisis periods (Xiao and Maillebuau, 2020; Wang and Huang, 2019).

The fourth type of empirical studies explore the relation between precious metals and country's macroeconomic indicators including GDP, Interest Rate, Exchange rate, Inflation amongst others. We have already stated earlier in the paper about the inflation hedge characteristics of the 'yellow metal'. Then impact of exchange rate change is also sometimes related to gold as a country'stend to use their gold reserves to stabilize their exchange rate. Then relation between precious metals especially gold and other commodities like crude and some metals (both ferrous and non-ferrous metals) has also been explored in many research studies.

The last dimension of research relates to understanding the co-movements of the precious metals amongst themselves. Here the researchers discussthe long and short run relation with long term adjustment (if any), spillover of return or volatility from one precious metal to another and also the lagged relation between these metals through causality relation. The findings here reveal that although a long term relation between gold and silver does exist, there are signs of weakening of this relation whenever economic conditionsundergo a change and again the same gets stronger after the crisis (Baur, and Tran 2014; Lucey and Tully 2006; Pradhan et.al. 2020).

On the other hand, some researchers have failed to detect any long term relation amongst the precious metals but a co-movement is witnessed for only few sub-periods. This was seen in a studies by Escribano and Granger, (1998); Wahab et al., (1994) amongst others. Escribano and Granger, (1998) in their study investigated the relation amongst gold and silver after the collapse of Bretton woods system of payments and found no long term relation amongst the variables. The sub periods of relationbetween the two was visible and this was coined the term 'Silver Bubble' and belonged to the period 1979-80.In another study Mishra, et.al. (2019) could prove that the relation amongst precious metals was only for short run with uni-directional causality moving from gold to silver indicating that investors in gold market could predict returns on silver markets. Schweikert, (2018) could find co-movement amongst gold and silver only during crisis periods. Using quantile cointegration framework they could confirm that the relation between two variables was non-linearwith asymmetry also seen amongst the variables. On the other hand, no cointegration during crisis and bubble periods but proving the same for entire period was seen in a study by Baur, and Tran (2014). They concluded that cointegration surely existed for periods when the property of store of value gains importance for investors. Thus, their overall conclusion was that cointegration between the metals was not stable.

Moving in the same direction, main objective of the present study is to determine whether or not, the two precious metals be considered as a single asset class and clubbed into one category by the investors and portfolio managers. This is broadly examined by analysing the co-movement between the two precious metals; gold and silver.A related objective is to determine whether the underlying objective is also relevant from Indian Perspective. Indian Perspective assumes importance as precious metals are known to enjoy considerable freedom in price determination in the country especially during the festive season when the demand for ornamentals is at its peak and all this happens in spite of a high correlation witnessed between international prices and Indian precious metal prices. The need for separate focus on Indian Market is also warranted by increased interest of the market participants in the while metal; Silver during the recent past. Yet another driving factor which necessitated such a research was to examine and compare the cointegration results by applying two different techniques; one linear and second non-linear when variables are proved to follow non randomness in their movement. The study also made an attempt to test for spillover of volatility from gold to silver and vice-versa using the technique of standardized residuals transmission as given by Masson, (1998) and also by Dungey and Martin, (2007). To achieve these objectives, the study collects daily closing price data on gold and silver from MCX exchange of India for tenyear period April 1, 2012-Mar 31, 2022.

The rest of the paper is structured as follows: Section 2 gives the Descriptive Statistics and Distribution Characteristics of two precious metals, Section 3 explains the methodology employed, Section 4 provides empirical results and finally we have Section 5 as conclusion and policy implications. The paper ends with references as Section 6.

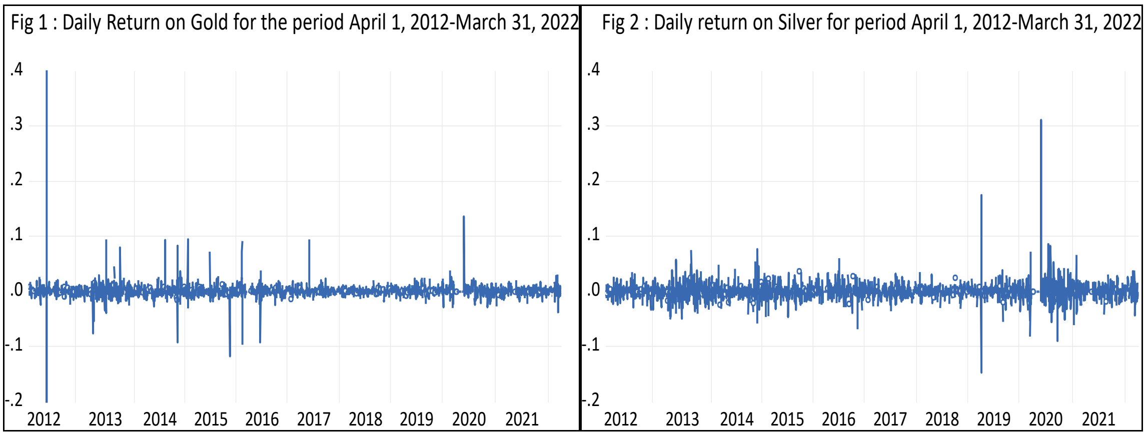
2. Statistical Description and Distribution Characteristics of Variables; Gold and Silver

Statistical Description of spot returns on two precious metals viz. gold and silver for the ten yearstudy period April 1, 2012-Mar 31, 2022 is presented in Table I below. The closing daily spot prices of these variables which have been collected from the www.mcxindia.com, have been transformed to daily spot returns by applying the formula; $\frac{P_t - P_{t-1}}{P_{t-1}}$, where $P_t$is the closing price of precious metal at day 't' and P_{t-1} is the closing price of the same precious metal at day 't-1'. The computation of returns from closing spot prices facilitates the comparison of precious metals across different parameters as shown under Table 1 below.

Table 1: *Statistical Data Description of Gold and Silver for the period April 1, 2012-March 31, 2022*		
Particulars	**Return on Gold**	**Return on Silver**
Mean	0.003457	0.000187
Std. Dev.	0.179584	0.015703
Coeff. of Variation (C.V) = σ / μ	56.67	18.36
Skewness	49.45623	3.249473
Kurtosis	2481.78	73.93063
Jarque - Bera Probability	6.52E+08 0.000	537779.4 0.000
Observation	2544	2544

Table 1 provides information about different parameters i.e. the four moments; Mean, Standard Deviation, Skewness and Kurtosis. The table also provides additional information on Coefficient of Variation (C.V) and Normality test for our two variables. The table reveals that mean return on gold spot is higher at 0.003457 on daily basis which translates to 126.18% on annualized basis while silver gives a spot daily return at 0.000187 or a mere 6.8 % return on annualized basis. Thus, in terms of return parameter and considering average returns per day as a yardstick, gold beats silver quite extensively over the stated ten year period. On the other hand, things are reversed when we consider second parameter; the standard deviation, which is considered a proxy for risk. Standard deviation of gold, is 0.17958which is approximately 11 times higher than that of silver having a standard deviation of 0.0157. The high variability or standard deviation of returns was also visible when we plotted the returns of the two variables (Fig 1 and 2) and it can be clearly seen that return variability of gold is far more than that of silver, however whether we also had volatility clustering along with high volatility would be known later when we test the two time series for heteroscedasticity.

Thus, from investors' point of view, although a lot of investors do consider gold as a safe haven asset which has already seen also gives a decent return, however over a ten year period of study, the results also revealed that it had a very high variability classifying this as a highly risky asset. This in simple words would mean that there are high chances that an investor might buy gold at high prices and then see a big fall in its prices within a short period of time. Thus, gold falls into high return and high risk category class of assets and hence the asset appears to be more suitable investment for seasonal investors who are also risk takers, but the same may not be true for a risk averse investor.



The statistical analysis of risk and return would be incomplete unless we evaluate the two assets by applying a measurement tool which would balance risk and return from an asset. To achieve this task, we compute another yardstick of measurement, the Coefficient of Variation (CV), which gives risk adjusted return for our asset under consideration. Now, lower the CV better is the risk adjusted return for precious metal under consideration. A look at the Table 1 reveals that return on gold has the CV of 56.67 and for silver it is 18.36 thereby making silver score over gold and making it a better choice for those investors who want to take calculated risk to obtain the desired return.

The other useful information available from Table 1 is about distribution characteristics of the two precious metals and these are the third and fourth moments i.e. Skewness and Kurtosis. The objective here is to compare these parameters with a normal distribution, which is symmetric and has a skewness of '0' and kurtosis as '3'. For our two distributions, although both distributions do not match the parameters of normal distribution, silver appears to be closer to normal than gold in terms of shape, peakedness and fatter tail characteristics. The examination of the two distributions reveal that both the distributions are positively skewed and leptokurtic Thetypical characteristic of a leptokurtic distribution is that these have a lot of outliers (extreme observations) which may have a lot of bearing on the final results. Further, we performed a formal test of normality, JB test which has a Null as Normal Distribution. The applicable formula is $JB = \frac{n}{6}\{S^2 + \frac{1}{4}(K-3)^2\}$, where 'n' is the number of observations, 'S' is the Skewness of the distribution and 'K' is the Kurtosis and it was seen that in case of both distributions; Null was rejected (probability 'p' was 0.00 for both distributions of precious metals)

3. Research Methodology and test of hypothesis

Statistical Distribution of Returns as given above has provided some insight about the return and risk characteristics of our two precious metals and the differences that are visible gives us an indication that the two assets need not be moving in the same direction at all times which gives us a rough idea that the two assets may be considered as separate assets to meet the goals of asset allocation and diversification. However more to obtain more insight into this aspect wemust examine this aspect from short and long run perspective and to this end we perform the cointegration and causality tests on the two variables. We begin with the test of cointegration for which we have applied two techniques which are discussed in next sub section.

3.1. Developing a Co-integration Model

Co-integration implies a balance between variables of the model in the long run. The need for this test arises as on many occasions a variable may tend to exhibit an inconsistent behaviour in the short run, however in the long run the same variable may tend to comove with other variables in a stable and predictable manner.

Since the objective of the study is to determine whether or not the two precious metals should be considered as a single asset class or two separate asset classes, it is imperative that we carry out a cointegration analysis to determine any long-term relation amongst the movement of these precious metals and to this end we would be applying two popular cointegration techniques, first one is the Johansen Co-integration Model (1998) and second beingGregory Hansen (GH) Co-integration Model (1996)which is sometimes called the Threshold Cointegration Model. Whereas, Johansen Co-integration Model (1998), although is a popular model as it is based upon a VAR, however the model assumes that the adjustment process from disequilibrium to long run equilibrium is symmetric or linear while the second model, GH Model can be applied in case the same follows non-linear character. The application of cointegration models, one which considers variables as linear and second as non linear is warranted by the main motive behind the present study. Here we would be first discussing the Johansen Co-integration followed by Threshold Cointegration Model.

3.1.1. The Johansen Co-integration Model (1998)

The Model estimates the following equation (1) and computes the ($\delta_{i,1}$-1) which is developed as a matrix of coefficients (γ_i).μ ismodel constant,'t' is the time period, 'i' is i^{th} variable, m denotes no. of lags of the variable Y_i, ΔY_t represents the short run relation with δ_1,δ_2, $\delta_3$as the slope coefficients and finallye_t is the error term.

$$\Delta Y_{it} = \mu + (\delta_{i,1}-1)(Y_{i(t-1)}) + \delta_{i,2}\Delta Y_{i(t-1)} + \delta_{i,3}\Delta Y_{i(t-2)} + \ldots + \delta_{i,k}\Delta Y_{i(t-(m-1))} + e_t \quad \ldots..(1)$$

Let γ_i signify long term relation among the variables; $\gamma_i$therefore is the fundamental matrix of the co-integration i.e.

$$\gamma_i = \begin{pmatrix} \beta_{11i} & \beta_{12i} & \cdots & \beta_{1mi} \\ \beta_{21i} & \cdots & & \vdots \\ \beta_{m1i} & \cdots & & \beta_{mmi} \end{pmatrix},$$

If there is no co-integration amongst any variables, $'\gamma'$shall have a rank '0' and in case of co-integration amongst variables, characteristic roots and Eigen values are computed. The popularity of Johansen model of co-integration lies due to the fact thatit avoids choosing a dependent variable and then subsequently running an OLS regression but this model treats every variable as endogenous variable. The method employs MLE Procedure which estimates the parameters by maximizing the likelihood function.

3.1.2. Threshold Cointegration Model [GH Model(1996)]

The second model of co-integration applied in our study is Threshold Cointegration Model or the Gregory Hansen (GH) Co-integration Model has a feature of testing the cointegration amongst variables in the presence of a structural break. Considering that our model is built up on high frequency daily data for ten year period, the chances of break in time series are therefore high, which actually necessitated the inclusion of thismodel in our study. In the presence of a break in time series, there is high probability that the cointegrating relation might undergo a change at the stated break point. We can imagine such a situation to be similar to somewhat a regime switch and the GH Model in a way cross checks and reinforce the results of Johansen Cointegration Model which ignores the structural break in time series. The Gregory Hansen test uses a simple methodology to incorporate a structural break by computing the usual ADF and Philips test statistics at all possible break points and then selecting the smallest values which becomes a breakpointfor the time series (Shahani and Singhal, 2022). However, the model also has its limitation in the sense that its inbuilt capacity to detect a structural break increases the chances of committing a type 2 error which actually would imply that rejection of Null of no co-integration becomes rather difficult (Shahani, Kumar and Goel 2020).

The paper discusses three variants of the Gregory – Hansen Co-integration Model viz. (i) Level shift (shift in intercept), (ii) Level shift with trend and (iii) Regime shift (or shift in both slope and intercept). The following equations (2, 3 and 4) are developed, one for each of three model versions:

Model I: Level Shift only: $Y_t = \theta_1 + \theta^*_1 D_{1,t} + \theta_2 X_{i,t} + e_{1t}$ .... (2)

Model II: Level Shift with trend: $Y_t = \theta_1 + \theta^*_1 D_{2,t} + \theta_2 X_{i,t} + \theta_3 T + e_{2t}$ …… (3)

Model III: Regime Shift: $Y_t = \theta_1 + \theta^*_1 D_3 + \theta_2 X_t + \theta^*_2 D_4 X_{i,t} + e_{3t}$ ……(4)

The breakpoint in Model I (eq.(2)) is represented by Dummy (D_1), which takes the following values$D_{1,t}$. $=\begin{cases} 1\ if\ t \geq BD_1 \\ 0\ if\ t < BD_1 \end{cases}$ i.e. Dummy shall be '0' if time period(t) is before the break date (BD_1) and shall be '1' if time period is after the break, including break date. BD_1 is the Break Date for Model I (see Table 3) and e_{1t} is the error term. For the model, intercept before the break shall be θ_1 while intercept after the break shall be $\theta_1 + \theta^*_1$. Further, in case of Model I, slope does not undergo any change.

Model II adds a trend variable 'T' to the Model I; thereby any variation in the variables due to trend is taken care of. The shift parameterθ^*_1*is* represented by Dummy (D_2) and is represented by$D_{2,t}$. $=\begin{cases} 1\ if\ t \geq BD_2 \\ 0\ if\ t < BD_2 \end{cases}$. BD_2 is the Break Date for Model II and e_{2t} is the error term.

Finally we have the Model III which is a Regime Shift Model where both intercept and slope undergo a shift. The shift in slope has been represented by a Dummy Variable (D_4)

and is the nature of multiplicative dummy when it get multiplied with the independent variable $X_{i,t}$. Thus, Model III includes two dummies; D_3 for a shift in intercept and D_4 for shift in slope. Both Dummies are defined alike i.e. Dummy ='1', if 't' ≥ Break Date and '0' otherwise (see Shahani, Kumar and Goel, 2020). The model has BD_3 and BD_4 as break date points.

Null Hypothesis (H_o) for all three Models: No Co-integration at break point.

Alternate Hypothesis (H_A): Co-integration exists at breakpoint.

The test criteria values for all the three models are directly taken from ADF (τ) & two Philip test statistics as $\mathbf{Z}_t(\tau)$ & $\mathbf{Z}_a$ (τ) and if all the three absolute computed valuesobtained from model viz. |ADF (τ)| , | Z_t (τ) | and |Z_a (τ)| are more than critical we reject the null hypothesis. The rejection of null implies that the linear combination of variables exhibit long run stable characteristics (co-integration) and the breakpoint included does not alter these characteristics.

3.2. Developing a Return and Volatility Spillover Model for Gold and Silver

Under this section we would be developing a return and volatility model and also testing for spillover from one precious metal to another. Volatility spillover (sometimes called volatility transmission) is the occurrence of volatility price change in one market causing its impact (usually a lagged one) on another market. We would be focusing on the technique of spillover which follows the standardized residuals transmission as given by Masson, (1998) and also by Dungeyand Martin, (2007). Under this technique we would be using the both GARCH (1,1) and GARCH (2,1) model and developing the conditional mean and variance equations, one each for the two precious metals namely gold and silver. Although GARCH (1,1) is popular model and is employed in most research studies, need for GARCH (2,1) becomes necessary to obtain squared residuals from the model to be used in variance equation.

3.2.1. Conditional Mean and Variance Equations

We start by developing equation (5) as given below which represents the conditional mean equation for variable gold has been developed as an AR(2) model with additional term as first lagged standardized residuals ($e_{(s),Silver,t-1}$) showing spillover in returns. The term ($e_{(s),Silver,t-1}$) have been obtained by first running AR(1) equation of Silver and then applying the formula $\boldsymbol{e_{(s),Silver,t-1}} = \frac{(e_{,Silver,t-1}) - \overline{(e_{,Silver,t-1})}}{\sigma_{(e_{,Silver,t-1})}}$ to obtain standardized residuals; where $\overline{(e_{,Silver,t-1})}$ is the mean of the residuals of the silver metal while $\sigma_{(e_{,Silver,t-1})}$ is the standard deviation of the residuals of the silver. We also have e_{gold_t} as the residual error term of eq (5). The variance equation under GARCH model eq.(5a) has a constant term ; α_1, the ARCH term ; $\alpha_2\, e^2_{gold,t-1}$ and a GARCH term; $\alpha_3 \sigma^2_{e_{gold_{t-1}}}$. The equation (5a) also includes additional termas first lagged standardized squared residuals of silver $\alpha_4 \left(e^2_{(s),silver,t-1}\right)$ showing spillover in return volatility, these have been directly obtained by running an AR(2) model for the variable silver and then following process of standardization as discussed above. Again using the same methodology we build up eq. (6) and eq.(6a) for our variable silver.

Spillover equation for Variable Gold

$$Y_{Gold,t} = \beta_1 + \beta_2 Y_{Gold,t-1} + \beta_3 Y_{Gold,t-2} + \beta_{4,Gold}\left(e_{(s),Silver,t-1}\right) + e_{gold_t} \text{.....eq. (5)}$$

$$\sigma^2_{e_{gold,t}} = \alpha_1 + \alpha_2\, e^2_{gold,t-1} + \alpha_3 \sigma^2_{e_{gold_{t-1}}} + \alpha_4 \left(e^2_{(s),silver,t-1}\right) \text{....eq. (5a)}$$

Spillover equation for Variable Silver

$$Y_{Silver,t} = \pi_1 + \pi_2 Y_{Silver,t-1} + \pi_3 Y_{Silver,t-2} + \pi_{3,Silver}\left(e_{(s),Gold,t-1}\right) + e_{silver_t} \text{.....eq. (6)}$$

$$\sigma^2_{e_{silver,t}} = \delta_1 + \delta_2\, e^2_{silver,t-1} + \delta_3 \sigma^2_{e_{silver_{t-1}}} + \delta_{4,m}\left(e^2_{(s),gold,t-1}\right) \text{....eq. (6a)}$$

3.3. Model Diagnostics

The models developed would give us robust results only when the model pre-requisites are satisfied to a reasonable extend. These are covered under Model Diagnosticsand here we state the methodology adopted for variable stationarity, serial correlation, stability of model and heteroscedasticity.

3.3.1. Stationarity

For stationary test we have applied Dickey Fuller Generalized Least Squares (DF GLS) technique which is known to have more power than simple ADF test. We have constructed eq. (7 and 8) to test stationarity of our variables, gold and silver.

$$\Delta\, \ddot{Gold}_t = \beta_1 \ddot{Gold}_{t-i} + \sum_{j=1}^{m} \beta_j\, \Delta\, \ddot{Gold}_{t-j} + e_{1_t} \ldots (7)$$

$$\Delta\, \ddot{Silver}_t = \alpha_1 \ddot{Silver}_{t-i} + \sum_{i=1}^{m} \alpha_i\, \Delta\, \ddot{Silver}_{t-i} + e_{2_t} \ldots (8)$$

In the above equations (7) and (8), $\ddot{Gold}$ and $\ddot{Silver}_t$ are the two de-trended variables with β_1 and α_1 being the two coefficients which test for the stationary of our variables. $\Delta\, \ddot{Gold}_{t-j}$ and $\Delta\, \ddot{Silver}_{t-i}$ are the augmentation terms which take care of serial correlation in time series and these are added 'm' times till serial correlation is removed. Further as we are working on de-trended data, model excludes intercept and time variable.

3.3.2. Serial Correlation

For serial correlation we apply BG-LM test and we develop an autoregressive regression for our variable gold as eq. (9) given as under:-

$$u_{Gold_t} = \beta_1 + \beta_2 Gold_{t-1} + \beta_3\, Gold_{t-2} + \ldots + \beta_p\, Gold_{t-p} + \rho_1 u_{Gold_{t-1}} + \rho_2 u_{Gold_{t-2}} \ldots + \rho_m u_{Gold_{t-m}} + e_t \quad (9)$$

('p' is the no. of lags in the regression and 'm' being the lags of the error term, BG-LM test assumes 'p' > 'm')

Null: $\rho_1 = \rho_2 = \ldots \rho_m = 0$ (no serial correlation between residuals). If R^2 (n-p) of eq. (9) > χ^2m, we reject the Null. Using similar methodology, we test our serial correlation for second variable silver.

3.3.3. Heteroscedasticity

For testing heteroscedasticity we apply Glejser (1969) method where the absolute residuals are regressed against the independent variable developed as three functional forms eq. (10a), (10b) and (10c). The equation with the highest R^2 and lowest standard error is selected and if slope coefficient of independent variable is significant, there is heteroscedasticity. The Null Hypothesis shall be Homoscedasticity.

E.g. for variable gold; $Gold_t = \beta_1 + \beta_2 Silver_t + e_{1_t}$ …(x)

$|e_{1_t}| = \beta_1 + \beta_2 Silver_t + e_{2_t}$…………….(x (a))

$|e_{1_t}| = \beta_1 + \beta_2 \sqrt{Silver_t} + + e_{2_t}$…………….(x (b))

$|e_{1_t}| = \beta_1 + \beta_2 \frac{1}{Silver_t} + + e_{2_t}$…………….(x (c))

3.3.4. Stability

Since we have applied VAR based cointegration model we check for stability of the parameters by applying Inverse Roots of AR Polynomial. In case of VAR Stability test, stability would exist only when all the characteristic roots lie in region ± 1.

3.3.5. Test for Linearity or the BDS test

BDS test statisticwas developed by Brock, et al., (1987) to detect a complex random non-linear pattern of variables and forms the basis for deciding the co-integration technique in our study. Under this we begin by specifying 'm' embedded dimensions ('m' histories and no. of observations 'n'> 'm') followed by Correlation Integral $C_{\varepsilon,m}$which measures spatial correlation between two points. We roll over the histories in the following manner: $y_1^m = y_1, y_2, y_3 \dots . y_m$, $y_2^m = y_3, y_4, y_5 \dots . y_{m+2}$ and so on.

We define our Null Hypothesis under BDS as follows:-

H_0: The data are independently and identically distributed (I.I.D.)

H_1: The data is not I.I.D. thus implying that the time series is non-linear

4. Results

Under this section we would be discussing the study results, the results of two co-integration tests, causality and volatility spillover results, followed by results of pre-requisites necessary for model building. (Table 2-9)

As already stated before, co-integration between gold and silver was tested by applying two models, Johansen (1998) Model and Gregory Hansen (GH) (1996) or Threshold Cointegration Model, the results of the same have been discussed in Table 2 and 3 respectively. The first Model i.e. Johansen (1998) test procedure uses the Trace test and Max Eigen value test with Null Hypothesis being stated in Column I as Hypothesized Number of Co-integrating relations in Table 2. Looking at the 'p' values obtained, we find the Null Hypothesis of No Co-integration is accepted using both Trace and Max Eigen Value Statistics thereby inferring that the test failed to detect any co-integration amongst the two precious metals.

Table 2: Johansen Co-integration results between GOLD and SILVER during the period (April 1, 2022-March 31, 2022)

Unrestricted Co-integration Rank Test (Trace & Max Eigen Value)

Hypothesized No. of CE(s)	Trace Statistic	Prob.	Max Statistic	Eigen Prob.
None	12.50791	0.1341	10.83655	0.1626
At Most 1	1.67136	0.1961	1.67136	0.1961

Notes (1) *The optimal lag has been identified as 5 using SC Criteria*

(2) The Table result shows no cointegration between the two variables

Source: Authors' own computation

The results of our second cointegration test, GH Co-integration test are shown under Table 3(a) pertaining to variable gold and 3(b) for variable silver. For each of these two tables we give results for three different versions; level shift, level shift with trend and regime shift. The results clearly show no cointegration amongst two variables for any of the three versions for both the tables showing that the two precious metals do not have a long run relation even if the assumption of linearity is relaxed (as in case of GH model).

Table 3 Gregory Hansen Co-integration resultsbetween Gold and Silver during the period (April 1, 2022-March 31, 2022)

Table 3(a) Gregory Hansen Co-integration: Dependent Variable : Gold

Model	Computed ADF('t')	Computed Z_a	Computed Z_t	Break Date according to ADF	Break Date according to Z_a	Break Date according to Z_t	Result
I: Level Shift	-4.519314	-49.3856	-3.922165	05-05-2012	05-02-2012	07-12-2012	No Cointegration
II: Level Shift with trend	-5.923038	-53.88832	-5.20849	14-07-2012	05-09-2012	18-04-2012	No Cointegration
III: Regime Shift	-6.183299	-74.63677	-7.663285	09-09-2012	07-05-2012	02-06-2012	No Cointegration

Table 3(b) Gregory Hansen Co-integration: Dependent Variable : Silver

Model	Computed ADF Further, ('t')	Computed Z_a	Computed Z_t	Break Date according to ADF	Break Date according to Z_a	Break Date according to Z_t	Result
I: Level Shift	-5.303465	-38.043211	-5.245075	06-02-2020	08-12-2020	06-03-2020	No Cointegration
II: Level Shift with trend	-5.429385	-52.705058	-5.626485	07-03-2020	6/21/2020	06-02-2020	No Cointegration
III: Regime Shift	-6.941655	-68.839119	-6.395135	7/19/2020	7/19/2020	7/19/2020	No Cointegration

Null Hypothesis : No Cointegration at breakpoint
Result : Null Hypothesis of No Cointegration is accepted

Further, GH test provides additional information about the breakpoint of the two precious metals and the results reveal that for Gold the breakpoint, broadly falls in the year 2012 while the same for silver the year is 2020.

Next, we discuss the results of our test of causality for which we have applied VAR Granger Causality Wald test (Table 4). An important consideration here is that although long run co-integration was not proved, short run causality amongst the precious metals still can exist and the study results also indicate short run causality moving from gold to silver (Null of No Causality is rejected in case of gold to silver). The reason identified for existence of short run causal behaviour but no long run relation amongst precious metals could be the tendency of investors to move away from stocks during adverse conditions in stock markets towards gold at first instance followed by silver thereby resulting in cause-effect relation between the two.

Further the causality results discussed also do match with the results of our next test which we carried out to check the spillover of return and volatility from one variable to another (Table 5). Thespillover test applied is based upon standardized residuals transmission as given by Masson (1998) and Dungeyand Martin, (2007) in their studies (for methodology see eq.(5)and eq.(6)). The results showed thatboth standardized residuals and standardized squared residuals from variable Gold are significant in the equation of silver showing that spillover exists from gold to silver both at return as well as volatility levels.

Table 4: VAR Granger Causality Wald test Results

Null Hypothesis	Chi- sq	Prob.	Degrees of freedom	Causality Direction
Gold does not cause Silver	11.1077	0.0493	5	Null Rejected, (Gold)→ (Silver)
Silver does not cause Gold	6.8321	0.2711	5	Null Accepted, (Silver)↛(Gold)

Our final set of resultspertain to that of Model pre-requisites which are given in Tables (6-9) and Fig (2). The first results are the BDS test results for linearity of variables of gold and silver respectively which are given under Table 6 (a) and 6(b) respectively.

These results reveal that at all different 'm' dimensions considered in the study, Null Hypothesis of linearity is rejected for both the variables Gold and Silver and this was one of the reasons behind inclusion of Gregory Hansen (1996) Cointegration Model which detects cointegration in the presence of a structural break. The next two tables (Tables 7 and 8) give the results for serial correlation and heteroscedasticity and both the tests accept the Null Hypothesis of no serial correlation and homoscedasticity which are model satisfactory.

We also carried out stationary test and test applied was DF-GLS test which is known to give a superior result than a simple ADF unit root test and here the results revealed that both variables stationary at I (1). Finally, we have results for stability of parameters (Fig 2) given as AR Characteristic roots polynomial. Here stability would exist only when all the characteristic roots lie in region ± 1 and we find that all the three dots on the plot are lying in region ± 1 showing that the model is stable.

Table 5: Spill-over of Return and Volatility from (a)Gold to Silver and (b) Silver to Gold (April 1, 2022-March 31, 2022)

	(a)Gold to Silver				(b) Silver to Gold		
Mean Equation	**Beta Coeff.**	**'p' value**	**Return Spill-over**		**Beta Coeff.**	**'p' value**	**Return Spill-over**
Std Residuals: Gold(-1)	0.00311	0.000	**Yes**		-	-	-
Std Residuals: Silver(-1)	-	-	-		0.00015	0.6133	No
Variance *Equation*	**Beta Coeff.**	**'p' value**	**Volatility Spill-over**		**Beta Coeff.**	**'p' value**	**Volatility Spill-over**
ARCH term	-0.01558	0.0496	NA		-0.156948	0.0001	NA
GARCH term	0.99362	0.000	NA		0.339367	0.0005	NA
Std Residuals SQR: Gold(-1)	-9.98E-05	0.0024	Yes		-	-	-
Std Residuals SQR: Silver (-1)	-	-			5.84E-03	0.2544	No

Table 6(a): BDS for Gold

Dimension	BDS Statistic	Prob.	Result
2	0.204817	0	Null Rejected, time series is non-linear
3	0.348536	0	Null Rejected, time series is non-linear
4	0.449188	0	Null Rejected, time series is non-linear
5	0.519598	0	Null Rejected, time series is non-linear
6	0.568745	0	Null Rejected, time series is non-linear

Notes : (1) Null Hypothesis: The data are I.I.D.
(2) Result : Null is Rejected

Table 6(b): BDS for Silver

Dimension	BDS Statistic	Prob.	Result
2	0.199829	0	Null Rejected, time series is non-linear
3	0.340506	0	Null Rejected, time series is non-linear
4	0.43897	0	Null Rejected, time series is non-linear
5	0.507626	0	Null Rejected, time series is non-linear
6	0.555288	0	Null Rejected, time series is non-linear

Notes: (1) Null Hypothesis: The data are I.I.D.
(2) Result : Null is Rejected

Table 7: BGLM Test results for Serial Correlation

Variable	**Obs. R Square**	**Prob. Chi Sq.(2)**	**Null Hypothesis**
Gold	14.553	0.321	Accept
Silver	21.765	0.127	Accept

Notes: (1) Null Hypothesis: Serial Correlation
(2) Result: Null is accepted

Table 8 :Glejser Heteroscedasticity Test

Variable	**Obs. R Square**	**Prob. Chi Sq.(1)**	**Null Hypothesis**
Gold	6.721	0.118	Accept
Silver	9.742	0.103	Accept

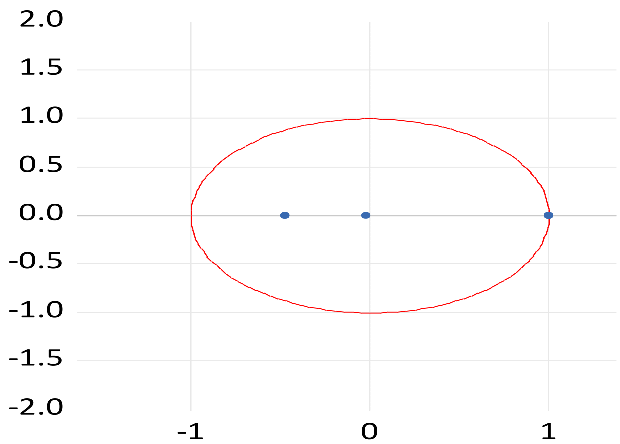
Notes: (1) Null Hypothesis: Homoscedasticity
(2) Result: Null is accepted

Table 9: Results of DF-GLS test for variable stationarity

Null Hypothesis	**Computed 't' statistics**	**DF-GLS test Critical Value at 5 %**	**Result**
Gold Spot Prices has a Unit root	-1.181889	-2.89	Null Accepted
Gold 1st Difference has a Unit root	-33.66941	-2.89	Null Rejected
Silver Spot Prices has a Unit root	-1.116159	-2.89	Null Accepted
Silver 1st Difference has a Unit root	-6.396205	-2.89	Null Rejected

Note: (1) Model includes trend and intercept', level of significance: 5%
(2) Result: Both Variables are stationary at first difference only

Fig 3 : Stability test : Inverse Roots of AR Characteristic Polynomial



5. Conclusion and Policy Implications

To conclude, the study made an attempt to examine whether the two precious metals viz. Gold and Silver are considered as single or two separate asset classes while considering key decisions pertaining to asset allocation and portfolio diversification. To achieve this objective, the paper first analysed the risk-return profile of the two assets and later developed a co-integration and volatility spillover model to understand the characteristics and dynamic movement of the two precious metals. Ten year daily closing prices for the period; April 1, 2012-March 31, 2022 was collected for the two metals from the MCX Exchange of India. The results of the study however failed to detect any long run cointegration for which two different tests were employed viz. Threshold and Johansen Co-integration Models. The need for two cointegration tests was necessitated after the BDS tests showed non linearity in the movement of both the variables under study. There can be two possible reasons for no long run cointegration amongst the precious metals, first in the long run as most financial markets tend to recover from recession or a crisis , investors who had shifted to safe havens like gold tend to revert back to traditional financial assets like stocks and bonds. Although same may also be true with silver, however as things are placed today, the asset still does not qualify as an appropriate safe haven asset in the eyes of investors and therefore question of reverting back from silver to other assets may not be in same proportion to gold which can impact the co-movement of two assets. The second reason why these two assets were not found to be co-integrated in the long run could be due to sharp differences in industry usage of two assets. Apart from being a hedge and safe haven, gold has a strong industry demand for ornamental purposes and as already seen before, 50 % of total global demand for this yellow metal comes only from ornamental industry. This is bound to impact the price of this asset and its correlation and co-movement with other financial assets.

In terms of other results, short run unidirectional causality was seen to move from gold to silver and also unidirectional return and volatility spillover was again seen moving from gold to silver from the results. The statistical description of data revealed vast differences in risk and return profile of the two assets. The model pre-requisites of Stationarity, Serial Correlation, Stability and Heteroscedasticity were tested and were found to be satisfactory. The study concludes that the two assets may be considered as two separate categories as no long run relation between the two variables was visible from results giving the portfolio managers a choice with respect diversification provided the viewpoint is of the manager is long run with respect to these assets.

The above results clearly give a signal that investors and portfolio managers could safely consider gold and silver as two separate asset classes and hold silver as well as gold in his/her portfolio as a part of asset allocation and diversification strategy where the viewpoint of such an investor is long run. Now considering the present dynamic economic and market conditions where almost all the assets have a tendency to move in tandem especially during a crisis situation thereby giving investors very little choice with respect to diversification, any new development in the financial markets like the one above which has any potential with respect to portfolio diversification is always a welcome sign.

Further, the unidirectional cause-effect relation from gold to silver would imply that lagged information ongold price movement could make a prediction about the price movement of silver giving an opportunity to the fund manager in the short run to develop a strategy to encash such a situation. e.g. if gold has seen an upside, the manager can expect the same for silver in near future and can easily sell gold and buy silver to benefit from the situation. Further the manager can work out similar mechanics with respect to volatility of two precious metals as spillover from gold to silver was also seen in the study results. Since both causality and volatility spillover is noticed to move from gold to silver, this also implies that the markets for gold and silver are ‘informationally’ inefficient in the short run and hence results are in line with some of the other research studies (Ntim, et al., 2015; Solt, et al., 1981).

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